Far-field intensity distribution of a beam generated by a resonator with a phase-unifying mirror

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The far-field intensity distribution (FFID) of a beam generated by a phase-unifying mirror resonator was investigated based on scalar diffraction theory. Attention was paid to the parameters, such as obscuration ratio and reflectivity of the phase-unifying mirror, that determine the FFID. All analyses were limited to the TEM$_{00}$ fundamental mode. © 2005 Optical Society of America

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1. Introduction
Phase-unifying (PU) mirrors have been widely used in optical resonators to generate laser beams with some special features.$^{1-4}$ The PU mirror has a kind of coating that does not affect the phase of the laser radiation, and it exhibits a stepwise reflectivity profile along its radius.$^5$ The PU mirror has a partial-reflection region in its center, surrounded by an antireflection region. The laser beam is reflected and magnified by the partial-reflection region and then propagates toward the antireflection region.$^6$ The structures of these two regions are specially designed such that the phases of the laser beam passing through them are unified. Moreover, unstable cavities with PU mirrors can produce beams of large mode volume with filled-in patterns.$^1$

The propagation of the beam can be regarded as a diffraction process. By studying the far-field problem we can learn some properties of the laser beam, such as its far-field intensity distribution and focusing properties.

The outline of this paper is as follows: Based on scalar diffraction theory, we deduce the far-field intensity distribution of a beam generated by a PU mirror resonator. Furthermore, an analytical solution of far-field distribution is given. Then the factors that influence the intensity distribution are discussed. All analyses are limited to a fundamental mode Gaussian beam (TEM$_{00}$).

2. Theoretical Analysis
A PU mirror gives a stepwise variable-reflectivity profile. A super-Gaussian mirror also exhibits a stepwise reflectivity profile along its radius if its order is $n \to \infty$.$^7$ Therefore a PU mirror can be equivalent to a super-Gaussian mirror of high order. The transverse profile of the lowest eigenmode is Gaussian inside a well-aligned resonator with a Gaussian mirror.$^8,9$ Then we consider the theory of a Gaussian beam inside a resonator with a PU mirror.

The geometry is illustrated and the main characteristics of the diffraction of a Gaussian beam by a PU mirror are shown in Fig. 1. We assume that the PU mirror’s radius satisfies the far-field diffraction condition and that there is no phase variation when a Gaussian beam passes through the PU mirror. Under these assumptions, we can obtain the complex amplitude distribution in the observation plane by integrating the Kirchhoff–Huygens formula$^{10}$

$$U(x_1, y_1) = \int \int U(x, y) \delta(x, y) \times \exp \left[ -\frac{i k}{z} (x x_1 + y y_1) \right] dx dy,$$

where $U(x, y)$ is the incident field. Let the incident
Gaussian beam be a TEM$_{00}$ fundamental mode with axial symmetry, which can be written as

$$U(x, y, z) = c \exp \left\{ - \frac{x^2 + y^2}{\omega^2(z)} \right\} \exp \left\{ -i \frac{z}{2R(z)} \right\},$$

(2)

where

$$c = \frac{\omega_0}{\omega(z)} \exp \left\{ -i \left[ k z - \arctan \left( \frac{\lambda z}{\pi \omega_0^2} \right) \right] \right\},$$

(2a)

$$\omega = \omega_0 \sqrt{1 + \left( \frac{z}{z_r} \right)^2},$$

(2b)

$$R(z) = z \left[ 1 + \left( \frac{z_r}{z} \right)^2 \right],$$

(2c)

$$z_r = \pi \omega_0^2 / \lambda.$$  

(2d)

Here $\omega_0$ is the Gaussian beam waist, $z$ is the distance from the waist to the diffracted screen, $z_r$ is the Rayleigh range, and $t(x, y)$ is the complex amplitude transmittance of the PU mirror, which can be expressed as

$$t(x, y) = \text{circ}(r/R) - \sqrt{R_0} \text{circ}(r/\varepsilon R) \quad (0 < \varepsilon < 1),$$

(3)

where $R$ is the radius of the PU mirror, $R_0$ is the reflectivity of the partial-reflection region, and $\varepsilon$ is the linear obscuration ratio in Eq. (3) (Fig. 1).

Referring to the polar coordinate systems in the phase-unifying mirror and the observation plane, respectively (Fig. 1), we can express Eq. (1) in the form of a Fourier–Bessel transform as follows:

$$U(\rho_1, \phi_1) = c \int_0^R \int_0^{2\pi} \exp \left\{ - \frac{\rho_1^2}{\omega^2} \right\} \text{circ} \left( \frac{\rho}{R} \right)$$

$$- \sqrt{R_0} \text{circ} \left( \frac{\rho_1}{\varepsilon R} \right) \exp \left\{ -ik \frac{\rho_1^2}{2R(z)} \right\}$$

$$\times \exp \left\{ - ik \frac{\rho_1}{z} \rho \cos(\phi - \phi_1) \right\} d\rho d\phi$$

$$= 2\pi c \int_0^R \exp \left\{ - \frac{\rho^2}{\omega^2} \right\} \text{circ} \left( \frac{\rho}{R} \right)$$

$$\times \exp \left\{ -ik \frac{\rho^2}{2R(z)} \right\} J_0 \left( \frac{k}{z} \rho \right) d\rho,$$

(4)

where $J_0$ is the zeroth-order Bessel function of the first kind. Then we can obtain

$$U(\rho_1, \phi_1) = 2\pi c \int_0^R \exp \left\{ - \frac{\rho_1^2}{\omega^2} \right\}$$

$$\times \exp \left\{ -ik \frac{\rho_1^2}{2R(z)} \right\} J_0 \left( \frac{k}{z} \rho \right) d\rho$$

$$- 2\pi c \sqrt{R_0} \int_0^R \exp \left\{ - \frac{\rho^2}{\omega^2} \right\}$$

$$\times \exp \left\{ -ik \frac{\rho^2}{2R(z)} \right\} J_0 \left( \frac{k}{z} \rho \right) d\rho. \quad (5)$$

For a simple description of Eq. (5) we introduce some parameters, given by

$$\frac{1}{u^2} = 1 - \frac{ik}{\omega^2},$$

$$v = \frac{k\rho}{z} = k\theta. \quad (6)$$

Hence Eq. (5) can be rewritten in the form

$$U(\theta) = 2\pi c \int_0^R \exp \left\{ - \frac{\rho_1^2}{u^2} \right\} J_0 (v\rho) d\rho$$

$$- 2\pi c \sqrt{R_0} \int_0^R \exp \left\{ - \frac{\rho^2}{u^2} \right\} J_0 (v\rho) d\rho. \quad (7)$$

Using the expression $(d/dx)[x^{n+1}J_{n+1}(x)] = x^{n+1}J_n(x)$, we can simplify Eq. (7):
According to the expression \( \lim_{x \to 0} \frac{2^n J_n(x)}{x^n} = \frac{1}{n!} \), the field at the center point in the observation plane can be expressed in the form

\[
U(\theta) = \pi R^2 c \exp \left( -\frac{R^2}{u^2} \left[ \frac{2^n J_n(kR\theta)}{(kR\theta)^n} \right] \right) - \pi \varepsilon R^2 c \frac{1}{\varepsilon R} \exp \left[ -\frac{(\varepsilon R)^2}{u^2} \right] \times \left[ \sum_{n=1}^{\infty} \frac{2^n J_n(k\varepsilon R\theta)}{(k\varepsilon R\theta)^n} \left( \frac{\varepsilon^2 R^2}{u^2} \right)^{n-1} \right].
\]  
(8)

An illustrative numerical example of intensity distribution is given in Section 3 below, where the parameter is \( n = 10 \) and wavelength \( \lambda = 1064 \) nm. The intensity distribution of the observation plane is described by

\[
I(p) = U(p)U^*(p),
\]  
(10)

where \( ^* \) represents the complex conjugate, with \( U(p) \) defined in Eqs. (8) and (9).

3. Results and Discussion

Figure 2 shows the far-field intensity distribution and the encircled power distributions, and the curves are plotted against the beam divergence angle normalized by \( \theta_0 \), which is defined by the angle at the first zero point of the Airy-disk pattern \( (\theta_0 = 1.22\lambda/2R) \). From Fig. 2 we can see that the intensity of the subdiffraction ring is much lower than that of the center diffraction ring. More than 90% of the energy is centralized, primarily into the center diffraction ring, with little energy diffracted into large angles with \( \varepsilon = 0.7 \), whereas, for a hard-edge mirror with \( \varepsilon = 1 \), less than 70% of the energy is within the diffraction limit. And the energy in the secondary diffraction rings is rather high. Therefore PU mirrors in optical resonators can improve beam quality.

In Fig. 3 we present the far-field encircled energy curves for the PU mirror at a variety of obscuration ratios with reflectivity \( R_0 = 0.5 \). When \( \varepsilon = 0.5 \), more than 80% of the energy is contained in the first ring. With an increasing obscuration ratio, improvement of
the intensity of the central disk is not apparent. Therefore, to improve the beam quality we suggest that the obscuration ratio of the PU mirror range from $\epsilon = 0.5$ to $\epsilon = 0.7$. Far-field encircled power distributions relative to a far-field angle are illustrated in Fig. 4 at reflectivities $R_0 = 0.1, 0.3, 0.5, 0.7, 0.9$ of the PU mirror with $\epsilon = 0.7$. From Fig. 4, the variety of intensity in the diffraction limit is less than 0.1 when the reflectivity ranges from 0.1 to 0.9. Hence we can see that improving the beam quality by varying the reflectivity does not produce better results than doing so by varying the obscuration ratio, whereas a comprehensive consideration of the output power and beam quality of the laser suggests that the reflectivity is $0.5–0.7$. The effect of $R_0$ and $\epsilon$ on the intensity distribution in the diffraction limit is clearly shown in Fig. 5.

4. Conclusions

Our objective has been to propose an analytical expression for the far-field intensity distribution of a Gaussian beam generated by a phase-unifying mirror resonator: First, we provided an accurate computation of the diffracted field and, second, we analyzed the parameters that determine the far-field intensity distribution, such as the obscuration ratio and reflectivity in the partial-reflection region of a phase-unifying mirror.

From the above discussion we can see that using a PU mirror as the output coupler of the resonator can improve beam quality. Ninety percent of the total energy of the PU beam is contained in diffraction limit, whereas less than 70% of the total energy of the hard-edge mirror beam is contained in the diffraction limit, and there are obvious diffraction rings in the far-field distribution of the hard-edge mirror diffracted beam. Therefore the use of a PU mirror in optical resonators can effectively restrain the expanding of far-field diffraction, substantially reduce the far-field divergent angle, and obviously eliminate diffraction rings in the far field. Besides, $\epsilon$ and $R_0$ can be adjusted to optimize the performance of the PU mirror, and a change in $\epsilon$ has a more apparent effect on the far-field intensity distribution than a change in $R_0$. In conclusion, the parameters $\epsilon = 0.7$ and $R_0 = 0.5–0.7$ can ensure good beam quality in theory.

References