Mutual alignment errors due to wave-front aberrations in intersatellite laser communications

Jianfeng Sun, Liren Liu, Maojin Yun, and Lingyu Wan

We analyze mutual alignment errors due to wave-front aberrations. To solve the central obscured problem, we introduce modified Zernike polynomials, which are a set of complete orthogonal polynomials. It is found that different aberrations have different effects on mutual alignment errors. Some aberrations influence only the line of sight, while some aberrations influence both the line of sight and the intensity distributions. © 2005 Optical Society of America


1. Introduction
Optical intersatellite links (ISL) provide an attractive alternate to microwave systems for both commercial and military applications. However, because of the small beam divergence and the ultralong distance of the link, the spatial-tracking requirement for an optical ISL is more stringent than that of a conventional microwave link. When the ISL is operated at such a narrow beam divergence (typically at 10µrad), the point error in the transmitter’s line of sight (LOS) can have a significant effect on the performance of the optical ISL. Mutual alignment errors are caused mainly by wave-front aberrations.

Optical devices for intersatellite laser communication systems are affected by many factors, such as the variation of the refractive index, the variation of the curvature of the lens surface, and the variation of the gaps between devices. The distortion of the optical devices can cause wave-front errors, which can change the far-field intensity distribution and mutual alignment errors. Toyoshima et al. have studied mutual alignment errors due to the variation of wave-front aberrations in a free-space laser communication link, but the transmitter for satellite laser communication systems regularly encounters centrally obscured pupils, such as in the case of a Cassegrain telescope system. In this case modal decomposition with Zernike polynomials no longer possesses the orthogonality property over the annular region.

In this paper we adopt a new series of polynomials by modifying the original Zernike polynomials by using the Gram–Schmidt orthogonalization procedure. Furthermore, the effects of the new series are analyzed.

2. Modified Zernike Polynomials
For each azimuthal frequency \( m \), the new radial functions \( Q_n^m(r) \) are related to \( R_n^m(r) \) by the following relation:

\[
R_n^m(r) = \gamma_{nm}^m Q_n^m(r) + \gamma_{nm+2}^m Q_n^{m+2}(r) + \cdots + \gamma_{nn}^m Q_n^n(r),
\]

(1)

where

\[
\gamma_{nm}^m = [2(n+1)]^{1/2} \left\{ \int_0^1 R_n^m(r) \left[ \sum_{j=0}^{n-m-2} \gamma_{nm+j}^m Q_n^{m+j}(r) \right]^2 rdr \right\}^{1/2},
\]

(2)

\[
\gamma_{nm+j}^m = 2(m+j+1) \int_0^1 R_n^m(r) Q_n^{m+j}(r) rdr,
\]

(3)

In Eqs. (2) and (3), \( \beta \) is the linear obscuration ratio and \( \gamma_{nm-2}^m \) is defined as zero. Obviously there exists an infinity of complete orthogonal sets because the
Gram–Schmidt orthogonalization process depends on the order in which the original Zernike radial polynomials are orthogonalized. From Eqs. (1)–(3) we know that when the obscuration ratio becomes zero, $Q_n^m$ degenerate to the conventional Zernike polynomials. Thus $Q_n^m$ may be viewed as a generalization of the conventional Zernike radial polynomials.

3. Line of Sight under the Condition of Aberrations

A. Direction of the Received Laser Beam

A complex amplitude of the wave $U_0(x_0, y_0)$ passing through a lens with a focal length $z = f$ is focused on the focal plane, and the optical field is given by

$$U_n(x, y) = \frac{A}{f} \exp\left[\frac{jk}{2f}(x^2 + y^2)\right] \int \int U_0(x_0, y_0) \times \exp\left[-j \frac{2\pi}{f}(xx_0 + yy_0)\right] dx_0 dy_0,$$

$$U_0(x_0, y_0) = B \exp[jk\Phi(x_0, y_0)],$$

where $k$ is the wave number, $\lambda$ is the wavelength, and $A$ and $B$ are constants that represent the strength or amplitude of the wave. Owing to the long distance between the two communication terminals, the received wave front can be considered as the plane wave. The definition of the coordinate systems is the same as in Ref. 9 (see Fig. 1). The intensity is then given by

$$I_n(x, y) = \frac{A^2}{\lambda^2 f^2} \left| \int \int U_0(x_0, y_0) \times \exp\left[-j \frac{2\pi}{f}(xx_0 + yy_0)\right] dx_0 dy_0 \right|^2.$$ (6)

Since a quadrant detector is used as the optical-tracking sensor in free-space laser communications, the center of gravity $(X, Y)$ of the received optical power on a quadrant detector aligned at $(x, y) = (0, 0)$, as shown in Fig. 2, is

$$X = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xI(x, y) dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(x, y) dx dy},$$

$$Y = \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} yI(x, y) dx dy}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(x, y) dx dy},$$

so the LOS is the vector $(X, Y, f)$.

B. Direction of the Transmitted Laser Beam

The laser beam transmitted from the telescope is generally the Gaussian beam, which can be written in the form

$$U_1(x_1, y_1) = \left(\frac{2}{\pi\omega_0^2}\right)^{1/2} \exp\left[-\frac{x_1^2 + y_1^2}{\omega_0^2} - j \frac{k(x_1^2 + y_1^2)}{2F_0} + j\Phi(x_1, y_1)\right].$$ (8)

The intensity distribution of the far-field can be written as

$$I_n(x, y) = \frac{A^2}{\lambda^2 f^2} \left| \int \int U_1(x_1, y_1) \times \exp\left[-j \frac{2\pi}{\lambda^2}(xx_1 + yy_1)\right] dx_1 dy_1 \right|^2.$$ (9)
Fig. 3. Effect of the tilt aberration \((Z2)\) on the receiver's intensity distribution: (a) \(x\) direction and (b) \(y\) direction.

Fig. 4. Effect of the defocus aberration \((Z4)\) on the receiver's intensity distribution: (a) \(x\) direction and (b) \(y\) direction.

Fig. 5. Effect of the astigmatism aberration \((Z5)\) on the receiver's intensity distribution: (a) \(x\) direction and (b) \(y\) direction.

Fig. 6. Effect of the coma aberration \((Z7)\) on the receiver's intensity distribution: (a) \(x\) direction and (b) \(y\) direction.
Fig. 7. Effect of the trefoil aberration ($Z_9$) on the receiver's intensity distribution: (a) $x$ direction and (b) $y$ direction.

Fig. 8. Transmitted intensity distribution without aberrations: (a) $x$ direction and (b) $y$ direction.

Fig. 9. Effect of the tilt aberration ($Z_2$) on the transmitted intensity distribution: (a) $x$ direction and (b) $y$ direction.

Fig. 10. Effect of the defocus aberration ($Z_4$) on the transmitted intensity distribution: (a) $x$ direction and (b) $y$ direction.
By using the same method, one can obtain the LOS of the transmitted beam.

C. Mutual Alignment Error Due to Wave Aberrations

The mutual alignment error is defined as the angle between the LOS of the received beam and the LOS of the transmitted beam. Next the mutual alignment error due to wave aberration is discussed. If the system were free of aberrations, the exit pupil would be filled by a perfect spherical wave converging toward the ideal image point. The aberration can be defined as the difference between the actual wave front and the Gaussian reference sphere. We use the modified Zernike polynomials to represent the aberration. From Eqs. (4)–(9), we can calculate the mutual alignment errors due to aberration.

In the calculation process, the parameters use the following values: $\gamma = 0.2889$, $\alpha = 1.579$, $\lambda = 847$ nm, and $D = 0.26$ m. To simplify the calculation, we assume that the curvature is infinity in the exit pupil plane. (Figures 2 and 8 give the received intensity and transmitted intensity distributions without aberrations, in which the intensity has been normalized.) In the calculations used to produce Figs. 2–13, each intensity distribution has been normalized with the maximum intensity of the received intensity without aberrations. According to the knowledge of the modified Zernike polynomials, $Z_1$ is equivalent to the uniform phase shift; it does not influence the received intensity distribution. After numerical anal-

Fig. 11. Effect of the astigmatism aberration ($Z_5$) on the transmitted intensity distribution: (a) $x$ direction and (b) $y$ direction.

Fig. 12. Effect of the coma aberration ($Z_7$) on the transmitted intensity distribution: (a) $x$ direction and (b) $y$ direction.

Fig. 13. Effect of the trefoil aberration ($Z_9$) on the transmitted intensity distribution: (a) $x$ direction and (b) $y$ direction.
ysis, we find that when the rms of aberration is less than $\lambda/10$, the effects of the aberrations can be ignored. To acknowledge the effect of the aberrations, we assume that the rms of each modified Zernike polynomial is equal to $\lambda/2$. Figures 3–13 give the effect of the different aberrations ($Z_2$, $Z_4$, $Z_5$, $Z_7$, and $Z_9$) on the received and transmitted intensity distributions from which the mutual alignment errors can be obtained. From the result we know that different aberrations cause different mutual alignment errors. The aberrations ($Z_2$, $Z_4$, $Z_7$, and $Z_9$) have slight influences on the LOS of both the receiver and the transmitter. The defocus aberration ($Z_4$) does not influence the LOS of the transmitter and the receiver, but it can cause a slight reduction of the power in the receiver and transmitter planes. The coma aberrations ($Z_7$ or $Z_9$) have the most important effects on the LOS and the intensity distributions. In this case the mutual alignment error caused by the coma aberration is 0.6222 $\mu$rad and the intensity reduces to 98% of the original value.

4. Conclusion
This paper analyzes the mutual alignment errors due to wave-front aberrations. To solve the central obscured problem, we introduce modified Zernike polynomials, which are a set of complete orthogonal polynomials. It is found that different aberrations have different effects on mutual alignment errors. Some aberrations influence only the LOS, while others influence both the LOS and the intensity distributions. This finding is very useful to the design of transmitters and receivers.

References